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## Modeling of reinforcement of reinforced concrete elements with composite materials

**Abstract.** One of the effective types of additional reinforcement of reinforced concrete elements is external with the use of composite materials. Known methods for calculating the parameters of the stress-strain state of reinforced elements are based on the general deformation and block models, which are applied separately without their interaction. For bending reinforced concrete elements reinforced under load in the tension zone, a method has been developed for calculating the parameters of their stress-strain state, taking into account the peculiarities of crack formation and the redistribution of forces between the main rod and additional external composite reinforcement at all stages of work up to failure. For bending reinforced concrete elements reinforced in the tension zone with external composite reinforcement, a model is proposed for taking into account tension concrete between cracks due to the application of additional stress in the reinforcement. Additional stresses are caused by the appearance of a difference in the relative deformations of the tensile reinforcement and concrete during the formation of a crack of normal separation. After strengthening the bending element in the absence of cracks of normal separation at the moment when the composite reinforcement is included in the joint work, additional external composite reinforcement takes part in the redistribution of forces between the concrete of the tension zone and the reinforcement. The degree of its participation is determined by the geometric parameters, the modulus of deformation and the law of adhesion to concrete. After reinforcement of bent reinforced concrete in the presence of cracks of normal separation (without injection during reinforcement), existing cracks continue to develop similarly to the first and second stages of cracking.

**Keywords:** reinforcement of reinforced concrete elements, composite materials, simulation of the stress-strain state, deformation diagrams, hypothesis of flat sections, general deformation model of calculation, law of adhesion of reinforcement to concrete, block model of calculation, calculation method, strength, crack resistance, deformability

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### Introduction

According to the results of verification calculations, the bearing capacity of curvilinear reinforced concrete elements is established, based on the actual technical condition and physical and mechanical properties of materials. When strengthening, when the destruction of the bent reinforced concrete element from the tension zone begins, additional reinforcement is installed in it. External reinforcement with composite materials is an effective type of additional reinforcement [1, 25]. Technologically, hardening is reduced to the formation of a multilayer composite material by gluing laminates, or layer-by-layer gluing of canvases, meshes with adhesives [2, 23, 24]. The task of external

reinforcement is to increase not only the bearing capacity of reinforced concrete elements (RCE), but also crack resistance, rigidity and durability of composite materials.

The basis for modeling the stress-strain state (SSS) of reinforced concrete elements when reinforcing them with composite materials are: general deformation model, hypothesis of flat sections (HFS); concrete and reinforcement deformation diagrams [2]. The distribution of relative deformations over the cross section has been refined. The additional composite material absorbs tensile forces when incorporated into the joint. The evaluation of its physical and mechanical characteristics is carried out according to the diagrams of its deformation. The situation is completely different when reinforcing reinforced concrete elements under load or after unloading. Indeed, at the time of combining into one structure with its additional composite reinforcement, reinforced concrete already has a certain SSS. It can be argued that in fact the reinforced structure becomes multi-component. It consists of used reinforced concrete, which is in a certain SSS, and the second component – composite reinforcement in its original state. Therefore, for reinforced RCE HFS for relative deformations is not performed. When the cross section is strengthened, some (residual) deformations from the impact of the load (unloading) already occur [3].

Before strengthening, it is necessary to take into account the initial SSS of the reinforced concrete structure. Relative deformations of the existing reinforcement and the most compressed concrete fiber are calculated for bending reinforced concrete elements in a simplified formulation. These relative deformations are then taken into account in the stress calculation based on HFS in the external composite reinforcement for the case of an over-reinforced element. In the non-linear deformation model (NDM) in [2], when calculating concrete before its strengthening, the relative deformations of the stretched face are calculated where the external composite reinforcement will be installed. Then additional composite reinforcement is taken into account in the calculation. The previously calculated initial relative deformations reduce the values of the limiting relative deformations of concrete under compression and tension. The values of limiting relative deformations of additional composite reinforcement included in the joint work increase. The position of the fiber of concrete or reinforcement along the height of the cross section and the load during reinforcement will be the determining factors for the limiting value of the relative deformations of concrete and reinforcement. Relative deformations of concrete and reinforcement before reinforcement according to the proposal [3] are calculated for all elementary areas of the calculated section. Then, after strengthening, they are used without limiting the relative deformations of concrete in compression and with limiting the relative deformations of concrete and reinforcement in tension.

In addition, in paragraph 6.1.5 [2] it is indicated that in the case of reinforcement under a load exceeding 65% of the design value, design characteristics of concrete and existing reinforcement should be multiplied by a reduction factor of working conditions equal to 0.9. The content of this paragraph [2] is in conflict with paragraph 6.28 [4], in which, under the above conditions, it is proposed to apply a reduction factor of working conditions equal to 0.9 for concrete and reinforced reinforcement. In accordance with the provisions of clause 6.28 [4], the Handbook [5] was developed.

The purpose of the research is to develop a methodology for calculating the parameters of their stress-strain state of bending reinforced concrete elements reinforced under load in a tension zone, taking into account the peculiarities of crack formation and the redistribution of forces between the main rod reinforcement and additional external composite reinforcement at all stages up to failure.

The objectives of this work are:

- develop a model for taking into account the work of concrete in a tensile zone between cracks of bending reinforced concrete elements reinforced with external composite reinforcement;
- to model the features of crack formation for bendable reinforced concrete elements reinforced under load in the tensile zone with a composite material, including the injection of normal tensile cracks.

### Stress-strain state of bending reinforced concrete elements, reinforced under load

The calculation of the SSS parameters of bent reinforced concrete elements reinforced with external composite reinforcement is carried out in two stages [2]. First, the system of equations of the general deformation model (GDM) is calculated for the calculated cross section of a reinforced concrete element bent in one plane. The system of equations consists of the equilibrium equations and the strain compatibility condition. The last condition describes the position of the strain distribution plane along the cross section in accordance with the hypothesis of flat sections (HFS) and has the form:

$$\begin{cases} \int \sigma_{c,s} A_{c,s} (y_{c,s} - y_0) dy = M; \\ \int \sigma_{c,s} A_{c,s} dy = 0; \\ \varepsilon_{c,s}^I = \frac{1}{r} (y_{c,s} - y_0), \end{cases} \quad (1)$$

where  $\sigma_{c,s}$  – normal stresses in the elementary area of concrete or reinforcement;  $M$  – bending moment from external influences when strengthening the element;  $\varepsilon_{c,s}^I$  – relative deformation in the elementary area of concrete or reinforcement;  $\frac{1}{r}$  – curvature of the element in the section under consideration when the element is strengthened;  $A_{c,s}$  – the cross-sectional area of the elementary area and concrete or reinforcement of a reinforced concrete element.

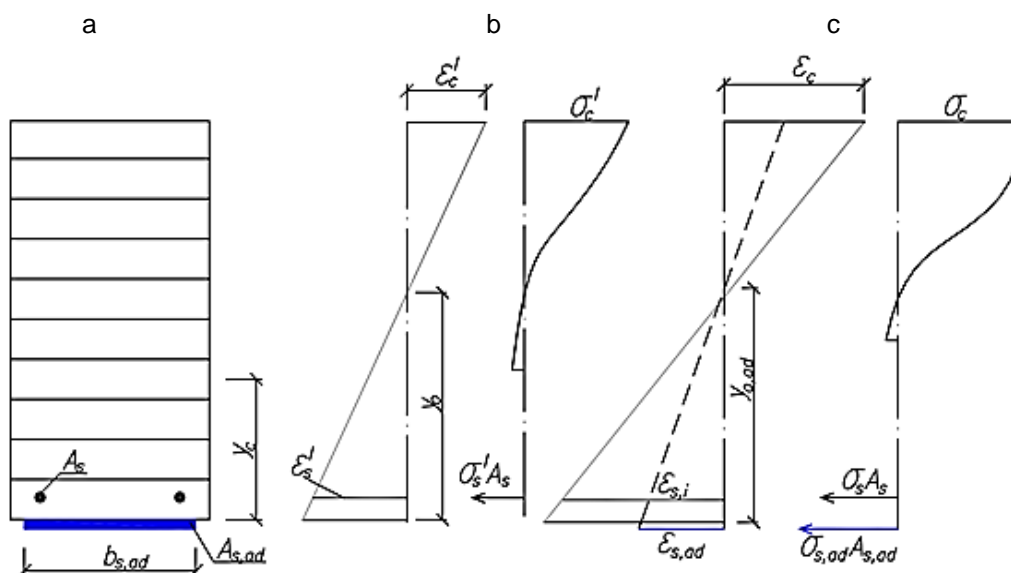
SSS equations for bent reinforced concrete element reinforced with external composite reinforcement, taking into account the presence of the main part of the reinforced element and additional composite reinforcement with rigid contact between them, the conditions for the balance of forces and the conditions for the compatibility of deformation for a reinforced cross section have the form:

$$\begin{cases} \int \sigma_{c,s} A_{c,s} (y_{c,s} - y_{0,ad}) dy + \sigma_{s,ad} A_{s,ad} (y_{s,ad} - y_{0,ad}) dy = M + \Delta M; \\ \int \sigma_{c,s} A_{c,s} dy + \sigma_{s,ad} A_{s,ad} dy = 0; \\ \varepsilon_{c,s} = \varepsilon_{c,s}^I + \frac{1}{r_{ad}} (y_{c,s} - y_{0,ad}); \\ \varepsilon_{s,ad} = \frac{1}{r_{ad}} (y_{s,ad} - y_{0,ad}), \end{cases} \quad (2)$$

where  $\sigma_{s,s}$  – normal stresses in composite reinforcement;  $\Delta M$  – increment of the bending moment from external influences after strengthening the element;  $A_{s, and d}$  – cross-sectional area of additional composite reinforcement of a reinforced concrete element;  $\varepsilon_{c,ad}$  – relative deformation of the composite reinforcement in the considered cross section after strengthening the element;  $\frac{1}{r_{ad}}$  – curvature of the reinforced element in the considered section from the action of  $\Delta M$ .

Relative deformations and stresses in the elementary areas of the reinforced element are calculated taking into account the approximations of the deformation diagrams of concrete, bar and composite reinforcement. The iteration process continues until the specified accuracy of the calculation is obtained under the conditions of the balance of efforts (2). Scheme of distribution of relative deformations and stresses in the cross section of a bent reinforced concrete element, reinforced with external composite reinforcement in the tension zone, shown in Fig. 1.

When calculating the corresponding load-bearing capacity of the reinforced element from the bending moment, the increment of the bending moment  $\Delta M$  increases in steps. Bearing capacity of the reinforced element  $M_{u,ad}$  corresponds maximum value of the bending moment ( $M + \Delta M$ ) from the external load, under which conditions (2) are satisfied.



**Fig. 1. Calculated section of bent reinforced concrete reinforced with external composite reinforcement in the tension zone (a) and distribution of relative strains and stresses before (b) and after (c) reinforcement**

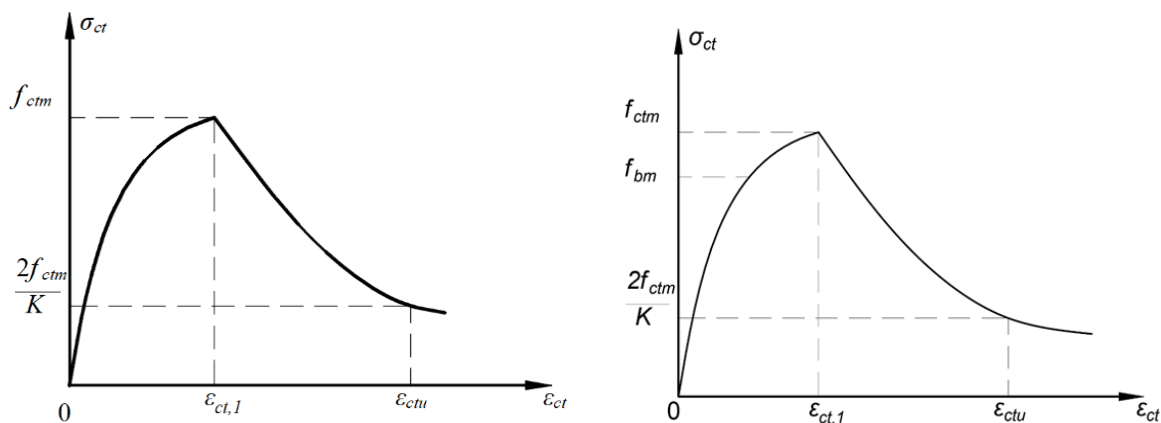
### Crack resistance of reinforced concrete elements

The crack resistance of reinforced concrete elements is characterized by the processes of formation and opening of cracks. The key parameters are the magnitude of the bending moment of crack formation and the width of their opening, taking into account the features of crack formation. The reinforced and reinforcing parts of the element are in different states. The reinforced part may have cracks normal separation in the stretched zone. When cracks are injected into the reinforced part of the reinforced concrete element, the concrete begins to work again in tension. The degree of loss of adhesion of the reinforcement to the concrete of the stretched zone along the length between the cracks depends on the level of stresses in the reinforcement and the nature of the acting load.

As a criterion for the formation of cracks of normal separation, the limiting relative deformations of the extreme stretched fiber are taken [6]. When calculating bending reinforced concrete structures, the tensile diagram of bar and composite reinforcement before it is crossed by a crack is taken as “free” reinforcement, and after crossing it, as a transformed one, taking into account the work of tensioned concrete between cracks [7].

The increment of the bending moment during the injection of cracks under load with polymer compositions (with the condition of their reliable adhesion with concrete) in the area adjacent to the crack, before the appearance of new cracks, is greater than the bending moment of the initial crack formation (Fig. 2). The effect is enhanced with an increase in the stresses in the reinforcement during injection and the time of their holding before injection, which is a consequence of the violation of the adhesion of reinforcement to concrete in the zones between cracks. In reinforced concrete with injected cracks, new cracks appear as a continuation of the original ones or in the immediate vicinity of the injected ones, when the adhesion of the polymer composition to concrete is less than its tensile strength.

When injecting initial cracks under load of bent reinforced concrete elements, the initial diagram of concrete tensile deformation is used in the calculation. In this case, the relative deformations are equal to their growth after crack injection. The value of the normal stresses of concrete is limited by the value characterizing the adhesion strength of the polymer composition with concrete  $f_{bm}$ . When performing nonlinear calculations of structures, the diagram of deformation of reinforced concrete under axial short-term tension takes on a curvilinear shape with a falling branch [8–10] (Fig. 2).



**Fig. 2. Diagram of concrete tensile deformation of reinforced concrete elements with injected cracks**

The ascending branch of the tensile reinforcement concrete diagram is described by the following relationship:

$$\sigma_{ct} = f_{ctm} \left[ 2 \left( \frac{\varepsilon_{ct}}{\varepsilon_{ct1}} \right) - \left( \frac{\varepsilon_{ct}}{\varepsilon_{ct1}} \right)^2 \right]. \quad (3)$$

To describe the descending branch (Fig. 2), the following dependence is accepted:

$$\sigma_{ct} = \frac{f_{ctm} \varepsilon_{ct1}}{\varepsilon_{ct}} = \frac{2f_{ctm} \varepsilon_{ctu}}{K \varepsilon_{ct}} = \frac{2f_{c,cube} \varepsilon_{ctu}}{K^2 \varepsilon_{ct}}, \quad (4)$$

where  $f_{c,cube}$  – concrete compressive strength, which is determined in cubes;  $f_{ctm}$  – tensile strength of concrete (stress at the top point of the deformation diagram);  $K$  is the ratio of the compressive strength of concrete to the tensile strength of concrete;  $\varepsilon_{ct1} = \frac{2f_{ctm}}{E_{ct}}$  is the value of the relative strain at the peak point of the strain diagram;  $\varepsilon_{ctu} = \frac{K \varepsilon_{ct1}}{2}$  – ultimate tensile strength of concrete;  $E_{ct}$  – is the initial modulus of elasticity of concrete.

The values of the coefficient  $K$  and the modulus of elasticity  $E_{ct}$  are determined by the formulas:

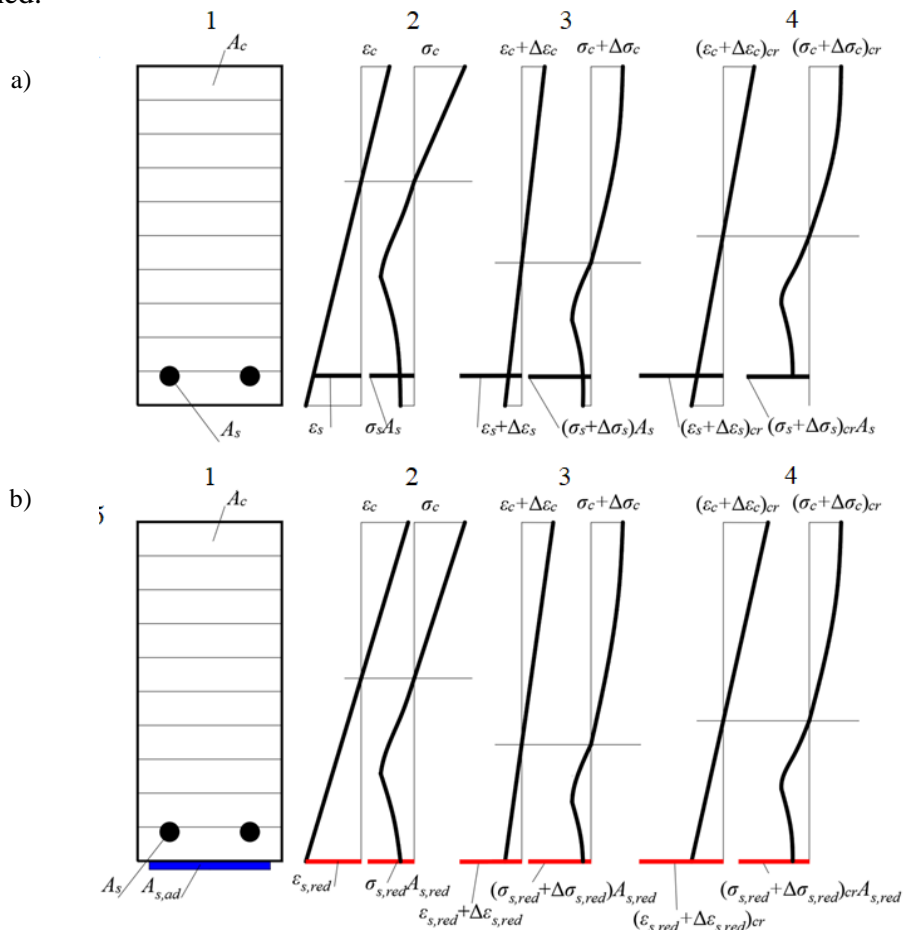
$$K = 6,4 + 0,1223 f_{cm}; \quad (5)$$

$$E_{ct} = \frac{10^7 f_{ctm}}{750 + 81,55 f_{ctm}}, \quad (6)$$

where  $f_{cm}$  is the average axial compressive strength of concrete, MPa ;  $f_{ctm}$  – average axial tensile strength of concrete, MPa.

Tensile strength of reinforced concrete elements for bending until cracks appear in concrete is provided by concrete and longitudinal reinforcement. The relative deformations of tensioned concrete and reinforcement are equal due to the adhesion between them. In the zone of maximum bending moments, the relative tensile strains in concrete approach the limit and cracks appear in the weakest places (due to the heterogeneity of the concrete structure and the variability of its resistance). In the section with a crack and near it, the cohesion is broken, and the tensile forces are perceived by the reinforcement. Stresses (relative deformations) after the formation of a crack along its edges in concrete become zero. There is a difference in the relative deformations of concrete and reinforcement  $\Delta\varepsilon$ . The difference in deformations as it moves away from the crack, according to the law of adhesion, decreases to zero or to the region where the relative deformations in concrete at the level of the center

of gravity of the reinforcement reach the limit values for tension and adjacent cracks appear. Additional stresses arise in the reinforcement due to the difference in the relative deformations of concrete and reinforcement after the transfer of forces from concrete. Additional stress varies along the length of the stretched concrete block depending on its value in the cross section with a crack and the value of the mutual shear of reinforcement and concrete. Additional stresses in the reinforcement, due to the difference in the relative deformations of the tensile reinforcement and concrete, form a new equilibrium state of the cross section. However, before and after crack formation normal separation, the bending moment from the external load in the cross section changes slightly. In bending reinforced concrete elements, this transition in the cross section due to the formation of a crack of normal separation occurs more smoothly, in contrast to centrally stretched ones. In [9] this phenomenon is called “additional condition”, in [10] – “compression damping in concrete”. The deformation model of deformation takes into account the effect of additional stresses in the reinforcement. This is implemented (similarly to modeling the prestressing of reinforcement [7]) (Fig. 3) by setting additional stresses (relative strains) to reinforcing bars. Additional relative deformations (stresses) in the reinforcement are calculated along the length of its shear section in the concrete of the tension zone. At the same time, according to the deformation model, a new equilibrium state of the section is taken into account (including sections with a crack), and the correspondence of the distribution of relative deformations of concrete and reinforcement along the height of the cross sections according to the HFS is established.



**Fig 3. Modeling the effect of additional stress in reinforcement on the equilibrium state of a section of a concrete block of a bending element without a crack:**  
**a, b – before and after reinforcement with external composite reinforcement; 1 – model of the cross section of the element; 2 – relative strains and stresses before the formation of a crack; 3 – the same after adding additional relative deformation (stress) from the mutual shear of concrete and reinforcement; 4 – after crack formation.**

**Block design model of a bent reinforced concrete element, reinforced with external reinforcement**

The SSS parameters of reinforced concrete between cracks are described in [11–15]. In addition to the characteristics of concrete, rod and composite reinforcement, the initial data are stresses (relative deformations) in the main  $\sigma_s$  and additional fittings  $\sigma_{s, ad}$  in a section with a crack, which are calculated for a given bending moment  $M$  (or  $M + \Delta M$ ) according to equations (1) or (2).

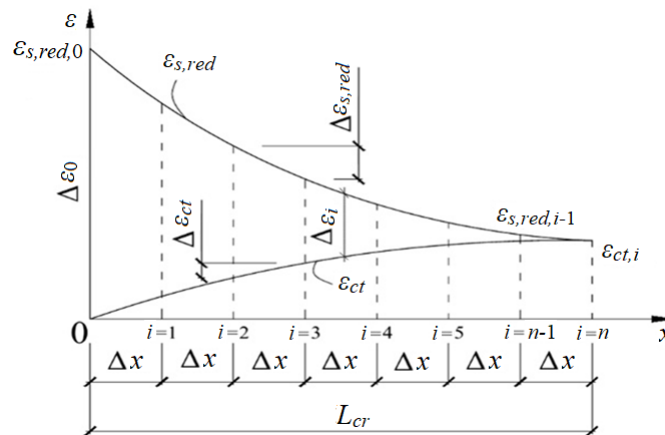
The first basic premise of the model [6]: a curved reinforced concrete element is a set of reinforced concrete blocks separated in the tension zone by normal separation cracks and interconnected by concrete of the compressed zone and tension reinforcement (steel rod and composite flat). The second basic premise is that the reinforcement and concrete of the tension zone work together in accordance with the laws of adhesion [16, 17], which establish the dependence of shear stresses on the contact area of the rod reinforcement with a diameter of  $\varnothing$  and external composite reinforcement with a thickness  $t_{ad}$  concrete from their mutual shift  $\tau(s)$  and  $\tau_{ad}(s_{ad})$ . The third basic provision: the stresses in the concrete of the tension zone are distributed uniformly over the effective area  $A_{c,eff}$ . For any block cross section, the total force  $N$  in tensile concrete  $N_c$ , bar reinforcement  $N_s$  and composite reinforcement  $N_{s,ad}$  constantly along the entire length of the block.

The distribution of relative strains of tensile concrete and reinforcement of the original (before reinforcement) bending element along the length of the concrete block is described by a system of equations. The solution is implemented by successive approximations of the finite difference method (Fig. 4):

$$\begin{cases} \frac{d}{dx} s = \varepsilon_s(\sigma_s) - \varepsilon_{ct} \left( \frac{N - \sigma_s A_s}{A_{c,eff}} \right); \\ \frac{d}{dx} \sigma_s = \frac{4}{\varnothing} \tau(s). \end{cases} \quad (7)$$

After strengthening the bending element, the system of equations for the distribution of relative strains of tensile concrete and reduced reinforcement along the length of the block has the form (Fig. 5):

$$\begin{cases} \frac{d}{dx} s_{s,red} = \varepsilon_{s,red}(\sigma_s, \sigma_{s,ad}) - \varepsilon_{ct} \left( \frac{N - \sigma_s A_s - \sigma_{s,ad} A_{s,ad}}{A_{c,eff}} \right); \\ \frac{d}{dx} \sigma_s = \frac{4}{\varnothing} \tau(s); \\ \frac{d}{dx} \sigma_{s,ad} = \frac{\tau_{ad}(s_{ad})}{t_{ad}}. \end{cases} \quad (8)$$



**Fig. 4. Distribution of relative deformations of concrete and reinforcement in a reinforced concrete block from the side of a crack in a bent reinforced concrete element, reinforced with external composite reinforcement in the tension zone**

Relative deformations of the reduced reinforcement  $\varepsilon_{s,red}$ , consisting of steel rod and external composite reinforcement, at the level of the most stretched face of the bent reinforced concrete element used in the system of equations (8), are calculated by the formula

$$\varepsilon_{s,red}(\sigma_s, \sigma_{s,ad}) = \frac{(\sigma_s A_s + \sigma_{s,ad} A_{s,ad})^2}{(\sigma_s A_s E_s + \sigma_{s,ad} A_{s,ad} E_{s,ad}) A_{s,red}}, \quad (9)$$

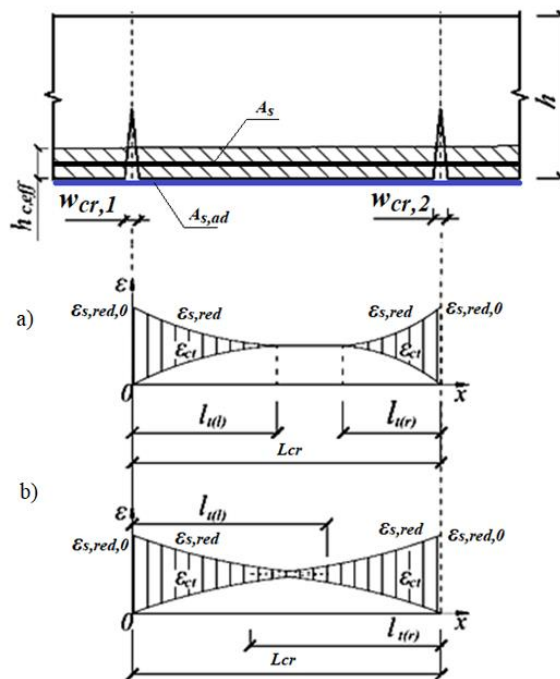
where

$$A_{s,red} = \frac{A_s E_s (y_s - y_{0,ad}) + A_{s,ad} E_{s,ad} (y_{s,ad} - y_{0,ad})}{E_s (y_s - y_{0,ad}) + E_{s,ad} (y_{s,ad} - y_{0,ad})}. \quad (10)$$

When operating a bent reinforced concrete element reinforced with external composite reinforcement in the tension zone, the following stages of cracking under load are possible. The first stage is unsteady cracking, when, with an increase in the bending moment along the length of the original reinforced concrete element, cracks of normal separation appear. At the second stage of steady-state cracking, when the number of cracks practically does not increase (there is a redistribution of forces between the reinforcement and concrete along the length of the reinforced concrete block in accordance with shifts due to adhesion failure).

At the first stage of crack formation in a reinforced concrete block, the relative deformations of concrete do not exceed the values of the limit concrete extensibility, i.e. the shear (redistribution) zones on the side of two adjacent fractures do not overlap (Fig. 5a). With an increase in the bending moment, the relative deformations of concrete at the level of the center of gravity of the reinforcement in the zone of joint deformation reach the values  $\varepsilon_{ctm,u}$ , new cracks of normal separation are divided into smaller reinforced concrete blocks.

At the second stage of crack formation along the entire length of the reinforced concrete block, the relative deformations of the concrete at the level of the center of gravity of the reinforcement  $\varepsilon_{ctm} \leq \varepsilon_{ctm,u}$ , i.e., the redistribution zones of two adjacent cracks overlap (Fig. 5b). With an increase in the bending moment from an external load, mutual shifts of reinforcement and concrete occur. This is manifested in the opening of cracks and the redistribution of forces from concrete to reinforcement.



**Fig. 5. Distribution of relative deformations of concrete and reinforcement along the length of the reinforced concrete block of the bent element after reinforcement with external composite reinforcement: a, b – at the first and second stages of cracking**



After strengthening the bending elements (in the absence of cracks normal separation at the moment when the external composite reinforcement is included in the joint operation), additional external composite reinforcement takes part in the redistribution of forces between the concrete of the tension zone and the reinforcement. The degree of its participation is determined by geometric parameters (width  $b_{ad}$ , thickness  $t_{ad}$ ), deformation modulus  $E_{s,ad}$  and the law of adhesion to concrete  $\tau_{ad}(s_{ad})$  [16–21]. Additional stresses in the reinforcement arise due to the difference in the relative deformations of the tensile reinforcement and concrete. They are applied at the level of the most stretched face of the bending element, to which the external composite reinforcement is glued (Fig. 4b).

After reinforcement of the bent reinforced concrete in the presence of cracks of normal separation (without their injection during reinforcement), the existing cracks continue to develop similarly to the first and second stages of cracking of the original bent reinforced concrete element.

The opening width of a crack of normal separation is calculated as the sum of mutual displacements  $s(x)$  of bar reinforcement and tensioned concrete (at the level of its center of gravity) in adjacent reinforced concrete blocks along the length of redistribution zones  $l_t$  to the left (l) and to the right (r) of the edges of the crack

$$w = \int_{-l_r(l)}^{l_r(l)} s(x) dx = \int_{-l_r(l)}^{l_r(l)} [\varepsilon_s(x) - \varepsilon_{cr}(x)] dx. \quad (11)$$

Coefficient  $\psi_s$  takes into account the influence of tensile concrete of a reinforced concrete block between cracks on the operation of reinforcement (for transform the diagram of its deformation after the appearance of cracks. It is calculated as the ratio of the area of the reinforcement stress diagram along the length to the area of the trapezoidal diagram with ordinates-stresses in cross sections with cracks at the ends of the reinforced concrete element.

Calculation of section stiffness along the length of a bent reinforced concrete element with cracks for calculating deflections is carried out taking into account the work of tensioned concrete as the average value of the sum of stiffnesses of elementary sections of concrete and reinforcement along the boundaries of the section

$$(EI)_i = \left( \left[ \sum E_{c,s} A_{c,s} (y_{c,s} - y_{0,ad})^2 \right]_i + [E_{s,ad} A_{s,ad} (y_{s,ad} - y_{0,ad})^2]_i + \right. \\ \left. + \left[ \sum E_{c,s} A_{c,s} (y_{c,s} - y_{0,ad})^2 \right]_{i+1} + [E_{s,ad} A_{s,ad} (y_{s,ad} - y_{0,ad})^2]_{i+1} \right) / 2 \quad (12)$$

## Conclusions

1. Based on the general deformation and block models, a method for calculating bending reinforced concrete elements, reinforced in the tension zone with external reinforcement with composite materials under load, is proposed. This makes it possible to obtain the parameters of their SSS at any stage of work under load for any combination of ultimate forces in rod and composite reinforcement, in any section, including the section between cracks.

2. For bending reinforced concrete elements reinforced in the tension zone with external composite reinforcement, a model is proposed for taking into account the work of tension concrete between cracks due to the application of additional stress in the reinforcement. Additional stresses are due to the difference in the relative deformations of the tensile reinforcement and concrete during the formation of a crack of normal separation.

3. When injecting normal separation cracks under load with polymer compounds (with the condition of their reliable adhesion to concrete) in the area adjacent to the crack, the increment in the bending moment after injection until new cracks appear is greater than the bending moment of formation of the initial cracks. The effect increases with increasing stresses in the reinforcement during injection and the time they are held before injection. If the adhesion of a polymer composition to concrete is less than its tensile strength in reinforced concrete elements with injected cracks, new cracks appear as a continuation of the original ones or in close proximity to the injected ones.

4. The degree of preloading before strengthening of bendable reinforced concrete elements reinforced with external composite reinforcement increases their deformability and the opening width of normal separation cracks.

Contribution of the authors: the authors contributed equally to this article.  
The authors declare no conflict of interests.

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*Строительные материалы и изделия*

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### **Моделирование армирования железобетонных элементов композиционными материалами**

**Аннотация.** Одним из эффективных видов дополнительного армирования железобетонных элементов является наружное армирование с применением композиционных материалов. Известные методы расчета параметров напряженно-деформированного состояния армированных элементов основаны на общих деформационных и блочных моделях, которые применяются отдельно, без их взаимодействия. Для изгиба железобетонных элементов, армированных под нагрузкой в растянутой зоне, разработана методика расчета параметров их напряженно-деформированного состояния с учетом особенностей трещинообразования и перераспределения усилий между основным стержнем и дополнительной внешней композитной арматурой на всех этапах работ вплоть до разрушения. Для изгиба железобетонных элементов, армированных в зоне растяжения внешней композитной арматурой, предложена модель учета растяжения бетона между трещинами за счет приложения дополнительных напряжений в арматуре. Дополнительные напряжения вызваны появлением разницы относительных деформаций растянутой арматуры и бетона при образовании трещины нормального отрыва. После усиления изгибаемого элемента при отсутствии трещин нормального отрыва в момент включения составной арматуры в стыковочную работу в перераспределении усилий между бетоном растянутой зоны и арматурой принимает участие дополнительная внешняя составная арматура. Степень ее участия определяется геометрическими параметрами, модулем деформации и законом сцепления с бетоном. После армирования изогнутого железобетона при наличии трещин нормального отрыва (без инъекций при армировании) существующие трещины продолжают развиваться аналогично первой и второй стадиям растрескивания.

**Ключевые слова:** армирование железобетонных элементов, композиционные материалы, моделирование напряженно-деформированного состояния, диаграммы деформирования, гипотеза плоских сечений, общая деформационная модель расчета, закон сцепления арматуры с бетоном, блочная модель расчета, метод расчета, прочность, трещиностойкость, деформируемость

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